An isogeny-based adaptor signature using SQISign

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University of Waterloo

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Payment Channel Networks

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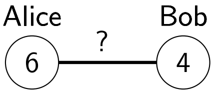

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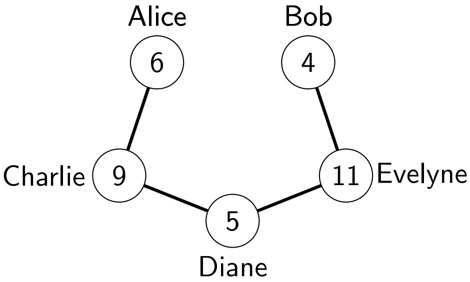

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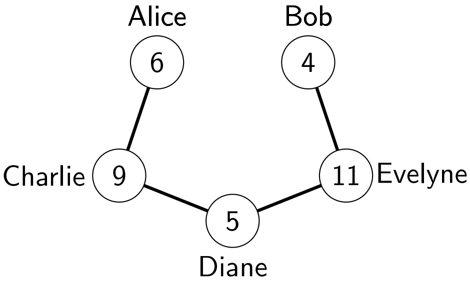

How can Alice be assured her money will arrive to Bob?

Anonymous Multi-Hop Locks (AMHL)

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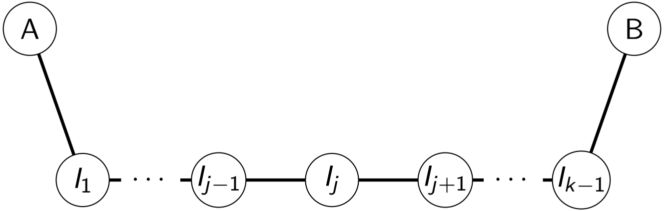

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She will then compute the following for each $j \in [1, \cdots k-1]$:

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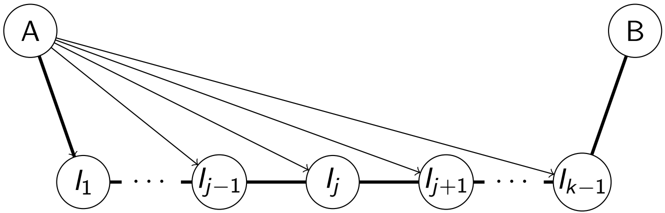

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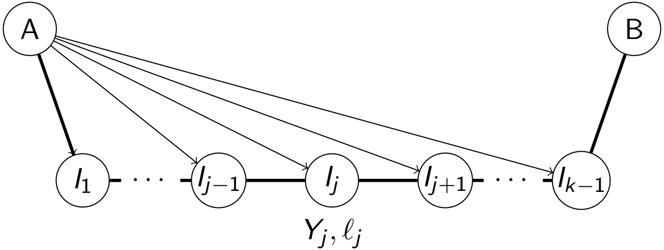

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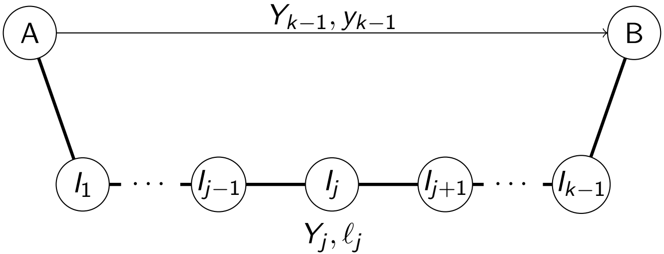

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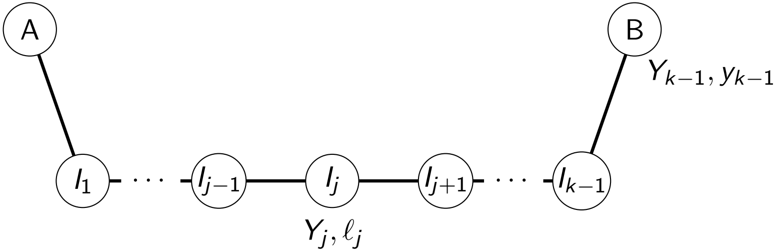

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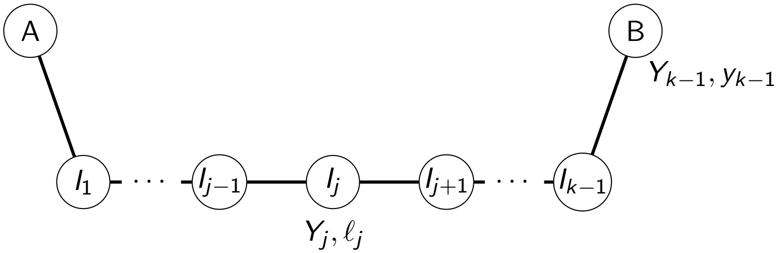

Intermediary I_j will sign a contract agreeing to release funds to I_{j+1} on the condition that I_{j+1} can provide y_j.

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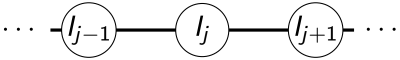

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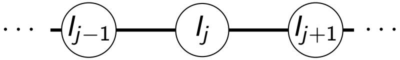

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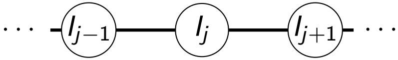

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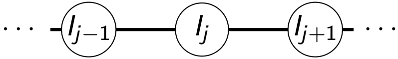

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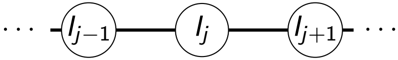

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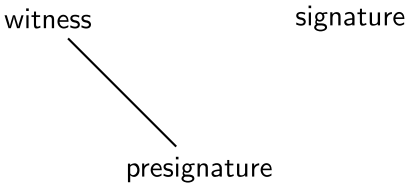
...how can we make this post-quantum?

Adaptor Signatures

witness signature

presignature

Adaptor Signatures



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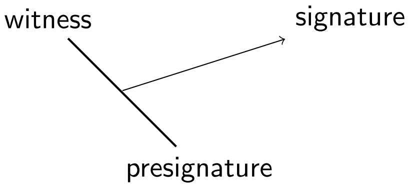

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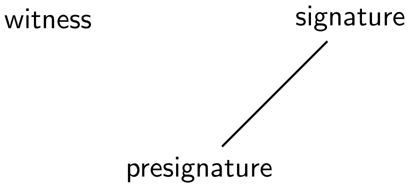

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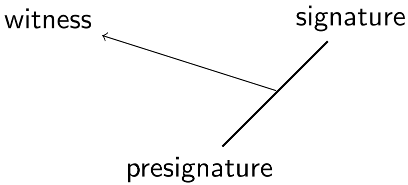

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Then an adaptor signature scheme with respect to R and Σ consists of four algorithms:

$$\mathsf{PreSig}(\mathsf{sk}, m, Y) \to \widetilde{\sigma}$$
$$\mathsf{PreVer}(\mathsf{pk}, m, Y, \widetilde{\sigma}) \to b \in \{0, 1\}$$
$$\mathsf{Adapt}(\widetilde{\sigma}, y) \to \sigma$$
$$\mathsf{Extract}(\sigma, \widetilde{\sigma}, Y) \to y$$

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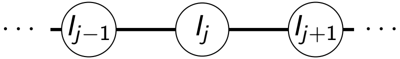

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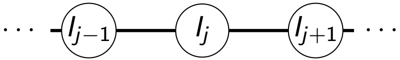

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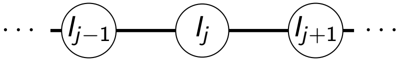

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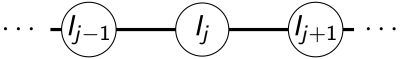

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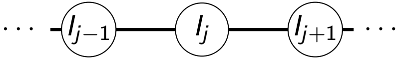

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Post-quantum adaptor signatures

Currently there are two post-quantum adaptor signature schemes:

- Lattice Adaptor Signature (LAS) using Dilithium (Esgin, Ersoy, Erkin, 2020).
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The latest generic construction (Dai, Okamoto, Yamamoto,2022) covers all signature schemes, but Extract and Adapt are trivial.

SQISign

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E_0

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$$E_0 \xrightarrow{\ \tau\ } E_A$$

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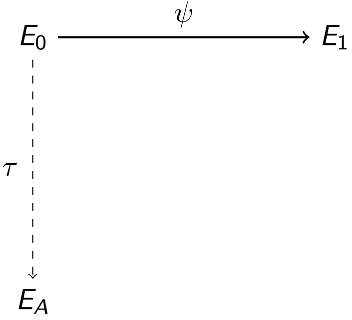

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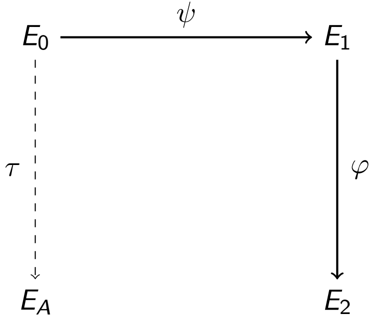

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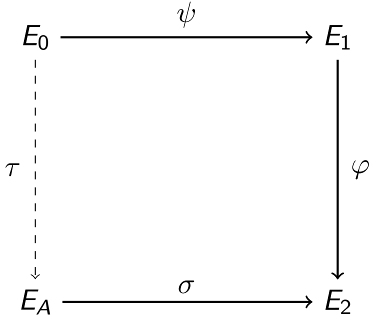

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Let (P_0, Q_0) be a basis for $E_0[\ell^e]$, for some small prime ℓ.
We choose our hard relation to be

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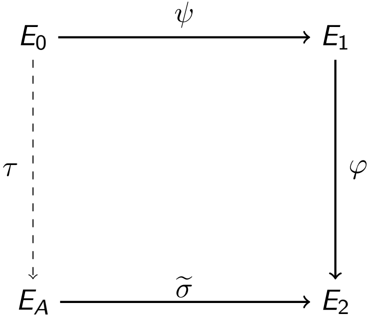

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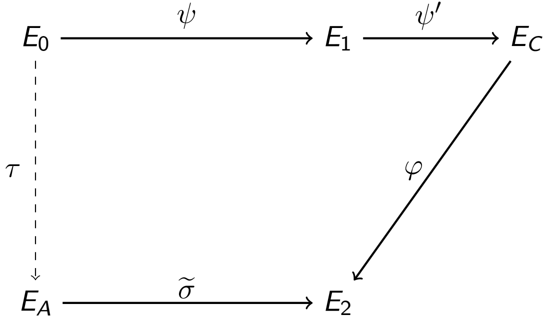

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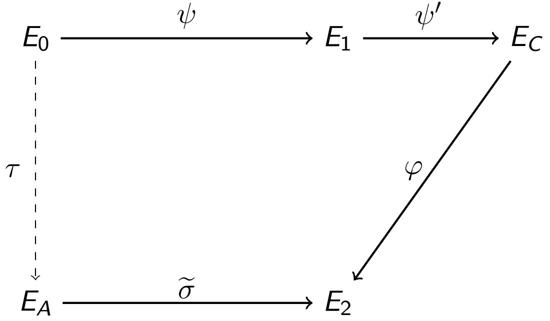

Include $\tau(P_0), \tau(Q_0)$ in PreSig

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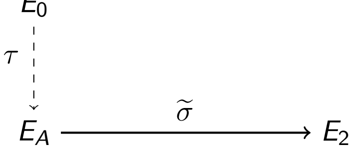

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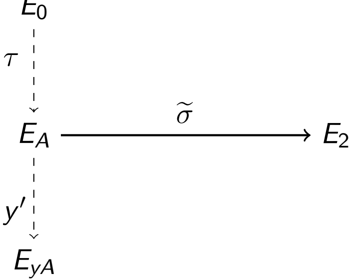

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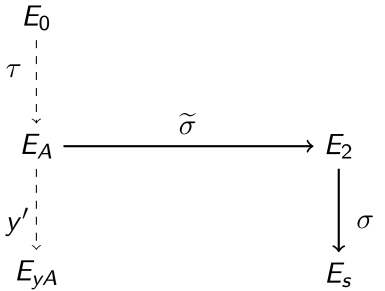
$$\sigma : E_2 \to E_s = E_2/\langle \widetilde{\sigma}(\tau(P_0) + \alpha_y \tau(Q_0)) \rangle$$



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Where it went wrong:

- Need a new prime;
- adhoc security assumptions;
- not secure.

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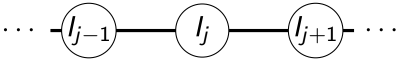
Anonymous Multi-Hop Locks (AMHL) via SAS

Release:



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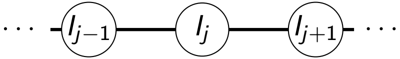

$$\sigma_j \leftarrow l_{j+1}$$

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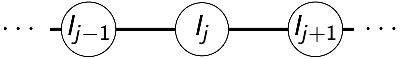

$$\sigma_j \leftarrow l_{j+1}$$

$$y_j \leftarrow \mathsf{Extract}(\sigma_j, \widetilde{\sigma}_j, E_{Yj})$$

$$\alpha_j \leftarrow y_j$$

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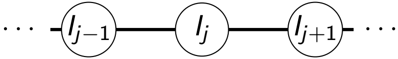

$$\sigma_j \leftarrow I_{j+1}$$

$$y_j \leftarrow \mathrm{Extract}(\sigma_j, \widetilde{\sigma}_j, E_{Yj})$$

$$\alpha_j \leftarrow y_j$$

$$\alpha_{j-1} = \alpha_j - \ell_j$$

$$\mathrm{y}_{j-1} : E_0 \rightarrow E_{Yj-1} \cong E_0/\langle P_0 + \alpha_{j-1} Q_0 \rangle$$

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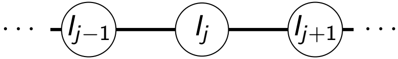

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$$\sigma_{j-1} \leftarrow \mathrm{Adapt}(\widetilde{\sigma}_{j-1}, y_{j-1})$$

Size Comparison in Bytes for 128-bit Security

	LAS	IAS	SAS
public key (bytes)	1472	128 - 2097152	64
presig (bytes)	2701	18327	226
sig (bytes)	3210	263 - 1880	15704

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The smaller presignature sizes in SAS make it better suited for *long* payment channel networks

- longer networks mean a longer set-up phase
- more will need to be transmitted to the participants